Automated Synthesis of Mechanisms

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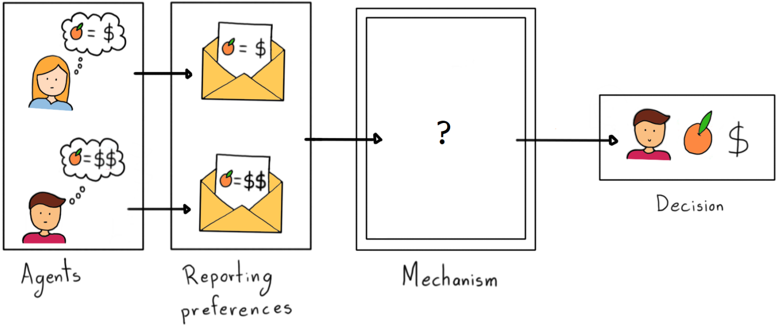
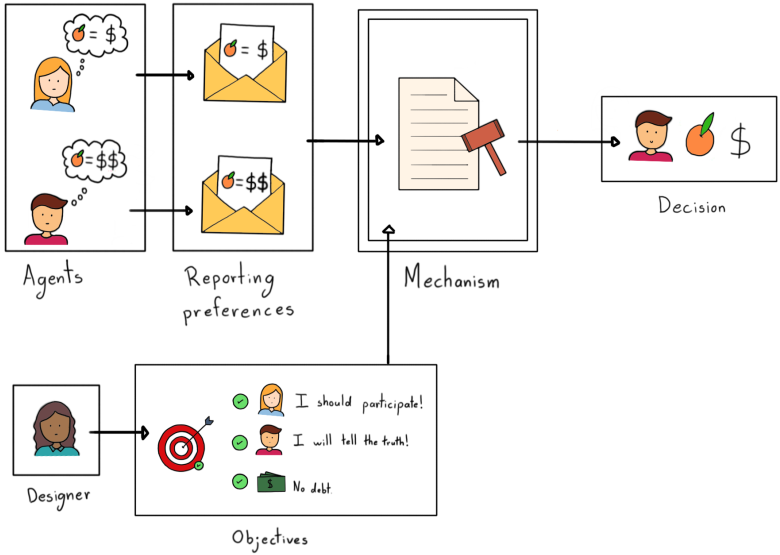
Automated Mechanism Design



Automated Mechanism Design



Automated Mechanism Design

- Automated Mechanism Design (AMD) (Sandholm 2003):
- Mechanism Design as a synthesis of Quantitative Strategy Logic (SL$[\mathcal{F}]$) formulas
- Specifications may involve requirements on the strategic behavior and the quality of the outcome

Quantitative Strategy Logic SL$[\mathcal{F}]$

- Weighted Concurrent Game Structure (wCGS) $\mathcal{G}$
- Syntax:

$$\varphi ::= p \mid \exists s.\, \varphi \mid (a, s_a)\varphi \mid f(\varphi, \ldots, \varphi) \mid \mathbf{X}\varphi \mid \varphi \mathbf{U} \varphi$$

  where f is a function in $\mathcal{F}$, p is a proposition, s_a is a variable, a is an agent
- Case where atomic propositions only take values in $\{-1, 1\}$ and $\mathcal{F}$ consists of $x \mapsto -x$ and $x, y \mapsto \max(x, y)$: SL

Solution concepts

- Nash equilibrium (NE)

$$\mathsf{NE}(\boldsymbol{s}) := \bigwedge_{a \in \mathsf{Ag}} \forall t. \left[(\mathsf{Ag}_{-a}, \boldsymbol{s}_{-a})(a, t)\mathbf{F}(\mathsf{util}_a) \leq (\mathsf{Ag}, \boldsymbol{s})\mathbf{F}(\mathsf{util}_a)\right]$$

Japanese auction

- $\mathbf{AG}((\neg \text{sold} \wedge \text{price} + inc \leq 1) \rightarrow (\text{price} + inc = \mathbf{X}\text{price} \wedge \neg \mathbf{X}\text{terminal}))$
- $\mathbf{AG}((\text{sold} \vee \text{price} + inc > 1) \rightarrow (\text{price} = \mathbf{X}\text{price} \wedge \mathbf{X}\text{terminal}))$
- $\mathbf{AG}(\text{choice} = \text{wins}_a \leftrightarrow \text{bid}_a \wedge \bigwedge_{b \neq a} \neg \text{bid}_a)$
- $\mathbf{AG}(\bigwedge_{a \in \text{Ag}} (\text{choice} = \text{wins}_a \rightarrow \text{payment}_a = \text{price}))$

Synthesis of mechanisms with SL$[\mathcal{F}]$

- Given a finite set $\mathcal{V} \subset [-1, 1]$ such that $\{-1, 1\} \subseteq \mathcal{V}$, the $\mathcal{V}$*-satisfiability problem* for SL$[\mathcal{F}]$ is the restriction of the satisfiability problem to $\mathcal{V}$-weighted wCGS.

> **Theorem (Satisfiability of SL$[\mathcal{F}]$)**
> The satisfiability of SL$[\mathcal{F}]$ is decidable in the following cases
> - wCGS with bounded actions
> - Turn-based wCGS

- Algorithms for the satisfiability problem of SL$[\mathcal{F}]$-> return a satisfying wCGS when one exists.

Optimal mechanism synthesis

Algorithm 1: Optimal mechanism synthesis

Data: A SL$[\mathcal{F}]$ specification Φ and a set of possible values for atomic propositions $\mathcal{V}$

Result: A wCGS $\mathcal{G}$ such that $[\![\Phi]\!]^{\mathcal{G}}$ is maximal

Compute $\widetilde{\mathsf{Val}}_{\Phi,\mathcal{V}}$;

Let $\nu_1, \ldots, \nu_n$ be a decreasing enumeration of $\widetilde{\mathsf{Val}}_{\Phi,\mathcal{V}}$;

for *i=1…n* **do**

  Solve $\mathcal{V}$- satisfiability for Φ and $\vartheta = \nu_i$;

  if *there exists* $\mathcal{G}$ *such that* $[\![\Phi]\!]^{\mathcal{G}} \geq \nu_i$ **then**

    return $\mathcal{G}$;

  end

end

Conclusion

- Logic-Based Mechanism Design
- Generating mechanisms $\rightarrow$ synthesis from SL$[\mathcal{F}]$-formulas
- Fragments of SL$[\mathcal{F}]$
- Probabilistic setting
  - Bayesian mechanisms
  - Mixed strategies
  - Randomized mechanisms

Thanks!

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